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TOPICAL REVIEW

Spin relaxation anisotropy in two-dimensional semiconductor systems

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Online at stacks.iop.org/JPhysCM/14/R271**Abstract**

Spin relaxation is investigated theoretically in two-dimensional systems. Various semiconductor structures of both n and p types are studied in detail. The most important spin relaxation mechanisms are considered. The spin relaxation times are calculated taking into account the contributions to the spin–orbit interaction due to the bulk inversion asymmetry and to the structure inversion asymmetry. It is shown that in-plane anisotropy of electron spin relaxation appears in III–V asymmetrical heterostructures. This anisotropy may be controlled by external parameters, and the spin relaxation times differ by several orders of magnitude.

1. Introduction

Spin dynamics in semiconductors has been investigated for more than four decades. During this period, effects leading to creation and disappearance of equilibrium and non-equilibrium microscopic magnetic moments have been discovered. Non-equilibrium spin is known to appear in III–V and II–VI compounds upon absorption of circularly polarized light. This way of creating non-equilibrium spins is named the method of *optical orientation* of electrons and nuclei, and has proved to be the most effective in investigating the optical and kinetic properties of bulk semiconductor samples and heterostructures. The problem of the loss of the average microscopic spin is very important in the analysis of experimental data and device applications.

In bulk semiconductors, mechanisms of electron and hole spin relaxation have been studied in ample detail both theoretically and experimentally [1]. It is known that the main mechanism for electrons in crystals lacking inversion symmetry is, for a wide range of temperatures, the kinetic mechanism proposed by D'yakonov and Perel'. Carriers lose their spin orientation owing to precession in the effective magnetic field caused by those terms which are cubic in wavevector. For holes in the valence band, where the spin–orbit interaction affects the energy spectrum to a greater extent, the loss of spin is mainly associated with a spin flip in each

scattering event (the Elliot–Yafet mechanism). However, the corresponding spin relaxation rate is of the same order as the momentum relaxation rate, which makes studying this kind of spin relaxation complicated.

In two-dimensional (2D) semiconductor structures, the relative importance of the above mechanisms changes dramatically because of the appearance of new forms of energy spectrum and spin–orbit interaction. The size-quantization effect suppresses the spin–orbit interaction for holes and increases its strength for electrons. Therefore, it becomes possible to study spin relaxation in p-type structures, and in n-type samples it is fundamentally different from that in the bulk case.

The spin dynamics processes in 2D systems may exhibit natural, ‘intrinsic’ anisotropy owing to presence of the growth axis. Analysis shows that in the conventional heterostructures this anisotropy is not large, being only 50%. However, the authors of the present study have shown theoretically that in-plane anisotropy of spin relaxation appears in III–V asymmetrical heterostructures [2]. This anisotropy may be controlled by external parameters, and the spin relaxation times differ by several orders of magnitude [3]. This opens up broad possibilities for spin engineering.

We consider the two most important carrier spin relaxation scenarios: the D’yakonov–Perel’ and Elliot–Yafet mechanisms. The Bir–Aronov–Pikus mechanism, associated with exchange electron–hole interaction, is essential only in p-doped samples, which are beyond the scope of this review, together with the excitonic spin relaxation.

In general, the spin relaxation equation can be written in the form

$$\dot{S}_i = -\frac{S_j}{\tau_{ij}}, \quad (1)$$

where S_i is the spin density, and $(1/\tau_{ij})$ is the component of a second-rank tensor. The concrete form of the tensor $(1/\tau_{ij})$ depends on the symmetry of the system under study. Since in a 2D structure a special direction exists (the growth axis), spin relaxation processes are anisotropic. The symmetry of a single heterointerface of a semiconductor with the zinc-blende lattice is C_{2v} . This leads to three different spin relaxation times. However, if two interfaces of a quantum well (QW) are equivalent, then the structure symmetry increases to D_{2d} and the in-plane spin relaxes isotropically. In semiconductors, spin relaxation processes are caused mainly not by an external magnetic field, but by peculiarities of the band structure and scattering mechanisms. These are the processes which are considered in the present review.

We write the electron or hole Hamiltonian in the form

$$H = H_0 + V \quad (2)$$

where H_0 is the Hamiltonian of a system without weak spin–orbit interaction and scattering, and these latter are described by the small perturbation V . Let $|n\mathbf{k}\rangle$ and $E_{n\mathbf{k}}$ be the eigenstates and eigenvalues of H_0 :

$$\langle n\mathbf{k}|H_0|n'\mathbf{k}'\rangle = E_{n\mathbf{k}}\delta_{nn'}\delta_{\mathbf{k}\mathbf{k}'}. \quad (3)$$

Here n is the index of a 2D electron or hole subband, and \mathbf{k} is the wavevector characterizing the motion in the heterostructure plane. In what follows, we assume that only the first subband of size quantization is populated and the energy dispersion is isotropic. Therefore, we omit the subband index and denote the energies as $E_{\mathbf{k}}$. It is convenient to extract the diagonal in the \mathbf{k} part of the spin–orbit interaction:

$$H_{\text{SO}}(\mathbf{k}) = V_{\mathbf{k}\mathbf{k}} \quad (4)$$

(small spin-independent corrections from $V_{\mathbf{k}\mathbf{k}}$ are assumed to be included in $E_{\mathbf{k}}$).

The dynamics of the electron spin density and that of the hole angular momentum density are determined by the time evolution of the respective density matrices. The diagonal in the \mathbf{k} -part of the density matrix, ρ_k , obeys the equation

$$\frac{\partial \rho_k}{\partial t} = \frac{i}{\hbar} [H_{\text{SO}}(\mathbf{k}), \rho_k] - \frac{2\pi}{\hbar} \sum_{k' \neq k} \left(\frac{V_{kk'} V_{k'k} \rho_k + \rho_k V_{kk'} V_{k'k}}{2} - V_{k'k} \rho_{k'} V_{k'k} \right) \delta(E_k - E_{k'}). \quad (5)$$

This equation is valid for any type of spin-orbit interaction and scattering. Below, we analyse it for the cases of electrons and holes separately.

The review is organized as follows. Section 2 is concerned with 2D electron systems. Section 3 is devoted to p-type quantum wells. The concluding section discusses the possible ways to measure spin relaxation rates and to observe the spin relaxation anisotropy.

2. Electron spin relaxation

A particular feature of electron systems is that the spin-orbit interaction is weaker than the momentum scattering. Therefore, it is convenient to represent the scattering amplitude as

$$V_{kk'} = \mathcal{I} u_{kk'} + V'_{kk'} \quad (6)$$

where \mathcal{I} is a 2×2 unit matrix and $\text{Tr}(V'_{kk'}) = 0$. Since the spin-orbit interaction is weak, $u \gg V'$.

In the following, we assume a central elastic scattering. In this case, $u_{kk'}$ is real and dependent on the electron energy E_k and the absolute value of the scattering angle $\theta = \varphi_{k'} - \varphi_k$, where φ_k is the angular coordinate of \mathbf{k} in the heterostructure plane.

Since $H_{\text{SO}}(\mathbf{k})$ and $V'_{kk'}$ are small perturbations, the spin relaxation times are much longer than the times in which the electron momentum distribution becomes isotropic. For this reason, it is convenient to represent the density matrix as a sum [1]:

$$\rho = \bar{\rho} + \rho', \quad \bar{\rho}' = 0.$$

Here the bar means averaging over φ_k and, hence, $\bar{\rho}$ depends only on E_k . The anisotropic part of the density matrix is due to H_{SO} and V' and, therefore, ρ' is small as compared with $\bar{\rho}$.

To second order in H_{SO} and V' , equation (5) has the form

$$\begin{aligned} \frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \rho'_k}{\partial t} = & \frac{i}{\hbar} [H_{\text{SO}}(\mathbf{k}), \bar{\rho}] + \frac{i}{\hbar} [H_{\text{SO}}(\mathbf{k}), \rho'_k] - \frac{2\pi}{\hbar} g \oint \frac{d\varphi_{k'}}{2\pi} |u_{kk'}|^2 (\rho'_k - \rho'_{k'}) \\ & - \frac{2\pi}{\hbar} g \oint \frac{d\varphi_{k'}}{2\pi} u_{kk'} \left(\frac{V'_{kk'} - V'_{k'k}}{2} \bar{\rho} + \bar{\rho} \frac{V'_{kk'} - V'_{k'k}}{2} \right) \\ & - \frac{2\pi}{\hbar} g \oint \frac{d\varphi_{k'}}{2\pi} u_{kk'} \left(\frac{V'_{kk'} + V'_{k'k}}{2} \rho'_k + \rho'_k \frac{V'_{kk'} + V'_{k'k}}{2} - V'_{kk'} \rho'_{k'} - \rho'_{k'} V'_{kk'} \right) \\ & - \frac{2\pi}{\hbar} g \oint \frac{d\varphi_{k'}}{2\pi} \left(\frac{V'_{kk'} V'_{k'k} \bar{\rho} + \bar{\rho} V'_{kk'} V'_{k'k}}{2} - V'_{kk'} \bar{\rho} V'_{k'k} \right) \end{aligned} \quad (7)$$

where g is the 2D density of electron states. It is noteworthy that the first two integrals in (6) are of first order in the spin-orbit interaction, and the two last are of second order.

The electron spin density is given by

$$S_e = g \int_0^\infty dE_k \text{Tr} \left(\bar{\rho} \frac{\boldsymbol{\sigma}}{2} \right). \quad (8)$$

Therefore, to analyse the spin dynamics, we have to find $\bar{\rho}$. Since $H_{\text{SO}}(\mathbf{k})$ and $V'_{kk'}$ are anisotropic in \mathbf{k} -space, the equations for $\bar{\rho}$ and ρ' are coupled and can be solved only for concrete interactions H_{SO} and V' .

In semiconductor heterostructures, spin relaxation can be caused by either H_{SO} or V' . The first scenario is known as the D'yakonov–Perel' mechanism, and the second as the Elliot–Yafet mechanism. Even in the same heterostructure, the relative contributions of the mechanism may vary with such parameters as temperature or electron concentration. Since either of the spin relaxation mechanisms may be dominant, we investigate them separately.

2.1. D'yakonov–Perel' mechanism

Let us consider a system with spin–orbit interaction described by the Hamiltonian $H_{\text{SO}}(\mathbf{k})$. It is equivalent to a Zeeman term with the effective magnetic field dependent on \mathbf{k} . In the presence of scattering, the wavevector changes and, hence, the effective magnetic field changes too. Therefore, in the case of frequent scattering, electrons are subjected to a chaotically changing magnetic field. The spin dynamics in such a system has diffusion character, which leads to loss of the specific spin orientation. This is the D'yakonov–Perel' spin relaxation mechanism [4], and it is the main spin relaxation mechanism in many III–V bulk semiconductors and heterostructures. It takes place even in the case of spin-independent scattering, when

$$V' = 0.$$

In this case, in a time of the order of the transport time τ_{tr} , the density matrix becomes isotropic, but there is no onset of spin relaxation processes. One can obtain a kinetic equation for the spin density at times longer than τ_{tr} [2]:

$$\dot{S}_{e,i}(t) = -\frac{1}{2\hbar^2} \sum_{n=-\infty}^{\infty} \frac{\int_0^{\infty} dE_k (F_+ - F_-) \tau_n \text{Tr}\{[H_{-n}, (H_n, \sigma_j)]\sigma_i\}}{\int_0^{\infty} dE_k (F_+ - F_-)} S_{e,j}(t). \quad (9)$$

Here

$$H_n = \oint \frac{d\varphi_k}{2\pi} H_{\text{SO}}(\mathbf{k}) \exp(-in\varphi_k) \quad (10)$$

are Fourier harmonics of the spin–orbit Hamiltonian,

$$\frac{1}{\tau_n} = \frac{2\pi}{\hbar} g \oint \frac{d\theta}{2\pi} |u_{kk'}|^2 (1 - \cos n\theta) \quad (11)$$

are the transport scattering rates, and $F_{\pm}(E_k)$ are the distribution functions of particles with spin projection equal to $\pm 1/2$.

Equation (9) is valid for 2D electrons with any spin–orbit interaction $H_{\text{SO}}(\mathbf{k})$. Let us consider an asymmetrical zinc-blende heterostructure. There are two contributions to $H_{\text{SO}}(\mathbf{k})$. The first, the so-called bulk inversion asymmetry (BIA) term, is due to the lack of inversion symmetry in the bulk of the material of which the heterostructure is made. To calculate this term, one has to average the corresponding bulk expression over the size-quantized motion [5]. We investigate the heterostructure with growth direction [001] coinciding with the z -axis and assume that only the first electron subband is populated. The BIA term has the form

$$H_{\text{BIA}}(\mathbf{k}) = \gamma[(k_z^2)(\sigma_y k_y - \sigma_x k_x) + (\sigma_x k_x k_y^2 - \sigma_y k_y k_x^2)] \quad (12)$$

where we choose the x - and y -directions aligned with the principal axes in the heterostructure plane. Here $\langle k_z^2 \rangle$ is squared operator $(-i\partial/\partial z)$ averaged over the ground state, and γ is the bulk spin–orbit interaction constant. It is seen that H_{BIA} contains terms that are linear and cubic in k .

In asymmetrical heterostructures, there is an additional contribution to the spin–orbit Hamiltonian, which is absent in the bulk. It is caused by structure inversion asymmetry (SIA) and can be written as [6–8]

$$H_{\text{SIA}}(\mathbf{k}) = \alpha(\sigma_x k_y - \sigma_y k_x) \quad (13)$$

where α is proportional to the electric field E acting on an electron:

$$\alpha = \alpha_0 e E. \quad (14)$$

Here e is the elementary charge and α_0 is a second spin-orbit constant determined both by the bulk spin-orbit interaction parameters and by the properties of heterointerfaces. It should be emphasized that, for asymmetrical heterostructures, E is mainly determined by the differences of the wavefunction and band parameters across the interfaces, rather than being the average electric field [9].

H_{SIA} contains terms linear in k , as also does H_{BIA} . From equation (9) it follows that the harmonics with the same n are coupled in the spin dynamics equations. This leads to interference of BIA and SIA terms linear in wavevector in the spin relaxation equations [2].

For $H_{\text{SO}} = H_{\text{BIA}} + H_{\text{SIA}}$, the system has C_{2v} symmetry. Therefore, equations (9) can be rewritten as follows:

$$\dot{S}_z = -\frac{S_z}{\tau_z}, \quad \dot{S}_x \pm \dot{S}_y = -\frac{S_x \pm S_y}{\tau_{\pm}}. \quad (15)$$

The times τ_z , τ_+ , and τ_- are the relaxation times of a spin parallel to the axes [001], [110], and $[1\bar{1}0]$, respectively.

If both the spin subsystems attain equilibrium before the onset of spin relaxation, then

$$F_{\pm}(E_k) = F_0(\mu_{\pm} - E_k) \quad (16)$$

where F_0 is the Fermi-Dirac distribution function and μ_{\pm} are the chemical potentials of the electron spin subsystems. If the spin splitting is small, i.e.

$$|\mu_+ - \mu_-| \ll |\mu_+|, |\mu_-|$$

then the expressions for the spin relaxation rates $1/\tau_i$ ($i = z, +, -$) have the form

$$\frac{1}{\tau_i} = \frac{\int_0^{\infty} dE_k (\partial F_0 / \partial E_k) \Gamma_i(k)}{\int_0^{\infty} dE_k (\partial F_0 / \partial E_k)} \quad (17)$$

where

$$\begin{aligned} \Gamma_z(k) &= \frac{4\tau_1}{\hbar^2} \left[(\gamma^2 \langle k_z^2 \rangle^2 + \alpha^2) k^2 - \frac{1}{2} \gamma^2 \langle k_z^2 \rangle k^4 + \frac{1 + \tau_3/\tau_1}{16} \gamma^2 k^6 \right] \\ \Gamma_+(k) &= \frac{2\tau_1}{\hbar^2} \left[(\alpha - \gamma \langle k_z^2 \rangle)^2 k^2 + \frac{1}{2} \gamma (\alpha - \gamma \langle k_z^2 \rangle) k^4 + \frac{1 + \tau_3/\tau_1}{16} \gamma^2 k^6 \right] \\ \Gamma_-(k) &= \frac{2\tau_1}{\hbar^2} \left[(\alpha + \gamma \langle k_z^2 \rangle)^2 k^2 - \frac{1}{2} \gamma (\alpha + \gamma \langle k_z^2 \rangle) k^4 + \frac{1 + \tau_3/\tau_1}{16} \gamma^2 k^6 \right]. \end{aligned} \quad (18)$$

Equations (17), (18) are valid for any electron energy distribution. If the electron gas is degenerate, then the spin relaxation times are given by

$$\frac{1}{\tau_i} = \Gamma_i(k_F) \quad (19)$$

where k_F is the Fermi wavevector determined by the total 2D electron concentration N :

$$k_F = \sqrt{2\pi N}. \quad (20)$$

In this case, the scattering time τ_1 in equations (18) coincides with the transport relaxation time τ_{tr} , which can be determined from the electron mobility.

For non-degenerate electrons, the spin relaxation times are determined, in particular, by the energy dependences of the scattering times τ_1 and τ_3 . If $\tau_1, \tau_3 \propto E_k^{\nu}$, then $\tau_3/\tau_1 = \text{constant}$

and

$$\begin{aligned} \frac{1}{\tau_z} &= \frac{4\tau_{tr}}{\hbar^2} \left[(\gamma^2 \langle k_z^2 \rangle + \alpha^2) \frac{2mk_B T}{\hbar^2} - \frac{\nu+2}{2} \gamma^2 \langle k_z^2 \rangle \left(\frac{2mk_B T}{\hbar^2} \right)^2 \right. \\ &\quad \left. + (\nu+2)(\nu+3) \frac{1+\tau_3/\tau_1}{16} \gamma^2 \left(\frac{2mk_B T}{\hbar^2} \right)^3 \right] \\ \frac{1}{\tau_{\pm}} &= \frac{2\tau_{tr}}{\hbar^2} \left[(\pm\alpha - \gamma \langle k_z^2 \rangle)^2 \frac{2mk_B T}{\hbar^2} + \frac{\nu+2}{2} \gamma (\pm\alpha - \gamma \langle k_z^2 \rangle) \left(\frac{2mk_B T}{\hbar^2} \right)^2 \right. \\ &\quad \left. + (\nu+2)(\nu+3) \frac{1+\tau_3/\tau_1}{16} \gamma^2 \left(\frac{2mk_B T}{\hbar^2} \right)^3 \right]. \end{aligned} \quad (21)$$

Here T is the electron temperature and k_B is the Boltzmann constant. In the particular case of short-range scattering, $\nu = 0$, and $\tau_1 = \tau_3$ are equal to τ_{tr} , which is independent of temperature.

The spin relaxation times are very sensitive to the relationship between two spin-orbit interaction strengths, $\gamma \langle k_z^2 \rangle$ and α . From equations (19), (21) it follows that at low concentration or temperature, $1/\tau_z$, $1/\tau_-$, and $1/\tau_+$ are determined by, respectively, the sum of squares $\gamma \langle k_z^2 \rangle$ and α , their squared sum, and their squared difference. This may lead to a considerable difference between the three rates, i.e. to a total spin relaxation anisotropy, if $\gamma \langle k_z^2 \rangle$ and α are close in magnitude. In real III-V systems, the relations between H_{BIA} and H_{SIA} may vary, with H_{BIA} or H_{SIA} being dominant [10, 11] or these two terms comparable [12]. In the latter case, a giant spin relaxation anisotropy is predicted [2].

The value of $\langle k_z^2 \rangle$ depends on the heteropotential and can be calculated for concrete asymmetrical heterostructures [3]. The constant γ is known for GaAs from optical orientation experiments [1]. Correct theoretical expressions for γ and α_0 have been derived in terms of the three-band $\mathbf{k} \cdot \mathbf{p}$ model [12, 13]. The heterointerfaces make a contribution to α_0 in addition to that from the bulk [14]. At large wavevectors, α_0 starts to depend on k [15, 16]. Here we assume that the concentrations and temperatures are sufficiently low, allowing us to ignore this effect.

In [3], the spin relaxation times were calculated for a III-V heterojunction and a triangular QW. The observance of spin relaxation anisotropy in all three directions is predicted for a wide range of structure parameters and temperatures.

It has been shown [17–19] that inclusion of both the BIA and SIA terms, equations (12) and (13), in H_{SO} leads to a spin-splitting anisotropy of the conduction band in \mathbf{k} -space in III-V semiconductor heterojunctions. However, the spin relaxation analysis performed in [18] ignored this effect. The authors of [20] demonstrated that the BIA and SIA terms interfere for weak localization but are additive for spin relaxation. It was shown in [2] that those terms in H_{SO} which are linear in the wavevector cancel out in spin relaxation as well.

In a recent study [21], the spin relaxation anisotropy was observed for uncommon (110) GaAs QWs. In this experiment, the spin relaxation in the growth direction was suppressed because of the ‘built-in’ anisotropy of the sample, resulting from the presence of heterointerfaces. By contrast, suppression of spin relaxation in the heterostructure plane was predicted in [2]. Moreover, all three spin relaxation times are different in the last case, and this effect takes place in ordinary (001) heterostructures.

We demonstrate that the terms in the spin-orbit Hamiltonian which are linear in the wavevector interfere, and this leads to a huge anisotropy of the spin relaxation times. At high concentration or temperature, this effect starts to disappear owing to the predominance of the cubic in k -terms in H_{SO} , which are only present in H_{BIA} . However, the higher-order terms in H_{SIA} are not forbidden by symmetry, either. These terms can also interfere with those in H_{BIA} , and cause additional non-monotonic features in the dependences of the spin relaxation times on the structure parameters.

2.2. Elliot–Yafet mechanism

Let consider a 2D electron system without spin splitting of the spectrum. In this case, there is no $H_{SO}(\mathbf{k})$ term in the Hamiltonian, and the spin–orbit interaction occurs due to scattering only.

In the heterostructures based on III–V semiconductors, the spin-flip scattering can be obtained in the Kane model taking into account a mixture of conduction band and valence band states. The electron wavefunction has the form [22]

$$\Psi_{s\mathbf{k}}(\mathbf{r}) = c_{\mathbf{k}} \exp(i\mathbf{k} \cdot \boldsymbol{\rho}) [u_s(z)S + v_{s\mathbf{k}}(z) \cdot \mathbf{R}]. \quad (22)$$

Here $s = +, -$ enumerates two spin states at a given \mathbf{k} ; S and $\mathbf{R} = (X, Y, Z)$ are, respectively, s- and p-like Bloch functions, and $c_{\mathbf{k}}$ is the normalization factor. The functions u_s and $v_{s\mathbf{k}}$ are eight envelopes corresponding to the conduction and valence band, respectively, related by

$$v_{s\mathbf{k}}(z) = i(A\mathbf{K} - iB\boldsymbol{\sigma} \times \mathbf{K})u_s(z). \quad (23)$$

Here we introduce a 3D vector $\mathbf{K} = (\mathbf{k}, k_z)$ with $k_z = -i\partial/\partial z$, and two constants:

$$A = \frac{P}{3} \left(\frac{2}{E_g} + \frac{1}{E_g + \Delta} \right) \quad B = -\frac{P}{3} \left(\frac{1}{E_g} - \frac{1}{E_g + \Delta} \right) \quad (24)$$

where E_g and Δ are the energy gaps between the bands Γ_6 and Γ_8 , and Γ_8 and Γ_7 , respectively, and P is the Kane matrix element. The envelope function for the conduction band is

$$u_s(z) = \varphi(z)w_s \quad (25)$$

where $\varphi(z)$ is the wavefunction of size quantization, and $w_+ = \uparrow$, $w_- = \downarrow$ are the spin functions. The normalization factor is given to second order in $A|\mathbf{K}|$ and $B|\mathbf{K}|$ by

$$c_{\mathbf{k}} = 1 - (A^2 + 2B^2)(k^2 + \langle k_z^2 \rangle) \quad (26)$$

where $\langle k_z^2 \rangle = \int dz (d\varphi/dz)^2$.

The scattering matrix element $\langle s\mathbf{k}|V|s'\mathbf{k}' \rangle$ is a 2×2 matrix with respect to the indices s and s' . Therefore, in the notation (6) we obtain

$$\begin{aligned} u_{\mathbf{k}\mathbf{k}'} &= u_0(\mathbf{k} - \mathbf{k}') [1 + (A^2 + 2B^2)(\mathbf{k} \cdot \mathbf{k}' - k^2)] \\ &\quad + (A^2 + 2B^2) \left[u_0(\mathbf{k} - \mathbf{k}') \langle k_z^2 \rangle - \int dz u_0(\mathbf{k} - \mathbf{k}', z) (k_z \varphi)^2 \right] \\ V'_{\mathbf{k}\mathbf{k}'} &= (2AB + B^2) \left\{ \int dz u_0(\mathbf{k} - \mathbf{k}', z) \varphi i k_z \varphi [\boldsymbol{\sigma} \times (\mathbf{k} + \mathbf{k}')]_z + i u_0(\mathbf{k} - \mathbf{k}') \boldsymbol{\sigma} \cdot (\mathbf{k} \times \mathbf{k}') \right\}. \end{aligned} \quad (27)$$

Here

$$u_0(\mathbf{q}, z) = \int d\boldsymbol{\rho} V(\boldsymbol{\rho}, z) \exp(i\mathbf{q} \cdot \boldsymbol{\rho}) \quad (28)$$

with $V(\boldsymbol{\rho}, z)$ the scattering potential, and

$$u_0(\mathbf{q}) = \int dz u_0(\mathbf{q}, z) \varphi^2(z). \quad (29)$$

The difference between $u_{\mathbf{k}\mathbf{k}'}$ and $u_0(\mathbf{k} - \mathbf{k}')$ is due to the spin–orbit interaction, similarly to the 3D case [23].

The two terms in V' are different: the part containing σ_x and σ_y is linear in k , but the term with σ_z is $\sim k^2$. Hence, at $k^2 \ll \langle k_z^2 \rangle$, the first part is dominant, and $V'_{\mathbf{k}\mathbf{k}'}$ can be represented as

$$V'_{\mathbf{k}\mathbf{k}'} = v(\mathbf{k} - \mathbf{k}') [\boldsymbol{\sigma} \times (\mathbf{k} + \mathbf{k}')]_z \quad (30)$$

where

$$v(\mathbf{q}) = (2AB + B^2) \int dz u_0(\mathbf{q}, z) \varphi i k_z \varphi. \quad (31)$$

It is noteworthy that a non-zero contribution at $\mathbf{k} = \mathbf{k}'$ is the impurity-induced SIA term, physically meaning the force exerted on electrons by scatterers. This term is to be added to H_{SO} , and it manifests itself in the D'yakonov–Perel' mechanism only.

It is seen from equation (30) that

$$V'_{kk'} = V'_{k'k}$$

for central scattering, whereas $v(\mathbf{k} - \mathbf{k}') = v(E_k, |\varphi_k - \varphi_{k'}|)$. Therefore, averaging of equation (7) over φ_k gives

$$\begin{aligned} \frac{\partial \bar{\rho}}{\partial t} = & -\frac{2\pi}{\hbar} g \oint \frac{d\varphi_k}{2\pi} \oint \frac{d\varphi_{k'}}{2\pi} \left\{ \frac{V'^2_{kk'} \bar{\rho} + \bar{\rho} V'^2_{kk'}}{2} - V'_{kk'} \bar{\rho} V'_{kk'} \right. \\ & \left. - u_{kk'} [(\rho'_{k'} - \rho'_k) V'_{kk'} + V'_{kk'} (\rho'_{k'} - \rho'_k)] \right\}. \end{aligned} \quad (32)$$

It can be shown that the term in square brackets is proportional to the integral

$$\oint d\theta u(\theta) v(\theta) \sin \theta$$

which is zero for central scattering.

Using equation (8), we obtain

$$\frac{1}{\tau_{zz}} = \frac{2}{\tau_{xx}} = \frac{2}{\tau_{yy}} = \frac{16\pi m}{\hbar^3} \frac{\int_0^\infty dE_k E_k [F_+(E_k) - F_-(E_k)] \oint d\theta v^2(E_k, \theta) (1 + \cos \theta)}{\int_0^\infty dE_k [F_+(E_k) - F_-(E_k)]}. \quad (33)$$

It can be seen that the Elliot–Yafet mechanism results in a 50% anisotropy of spin relaxation times.

To second order in Δ/E_g , we have for a Boltzmann gas

$$\frac{1}{\tau_{zz}} = \frac{32}{9} \left(\frac{\Delta}{E_g} \right)^2 \frac{k_B T}{E_g} r \frac{1}{\tau_{tr}} \quad (34)$$

with

$$r = \frac{\hbar^2 \int_0^\infty dE_k E_k \exp(-E_k/k_B T) \oint d\theta (1 + \cos \theta) [\int dz u_0(\mathbf{q}, z) \varphi(z) i k_z \varphi(z)]^2}{m E_g \int_0^\infty dE_k E_k \exp(-E_k/k_B T) \oint d\theta (1 - \cos \theta) [\int dz u_0(\mathbf{q}, z) \varphi^2(z)]^2}. \quad (35)$$

Here the wavevector transfer in the case of elastic scattering is described by

$$q = \sqrt{\frac{8mE_k}{\hbar^2}} \left| \sin \frac{\theta}{2} \right|$$

and account was taken of the fact that, at a given accuracy,

$$\frac{\hbar^2}{m} = \frac{P^2}{E_g}.$$

The factor r is of the order of E_1/E_g , where E_1 is the energy of the first level of size quantization. For an infinitely deep rectangular QW of width 100 Å, $E_g = 1.5$ eV, and for randomly distributed δ -scatterers, $r = 1.2 \times 10^{-2}$. Hence, for the parameters of GaAs ($\Delta = 0.34$ eV, $m = 0.067m_0$) and $T = 77$ K, we have $\tau_{tr}/\tau_{zz} \sim 10^{-5}$.

In addition to the scattering processes leading to spin relaxation considered above, there is a short-range interaction making possible scattering of electrons from the Γ_8 or Γ_6 band to the Γ_7 band [1]. The same mechanism takes place in the 2D systems as well [24].

3. Hole spin relaxation

A particular feature of p-type QWs is a strong spin–orbit interaction: the states at the top of the valence size-quantized subbands have a certain projection of the angular momentum on the growth axis. Therefore, ordinary spin-independent scattering leads to hole angular momentum relaxation even in a symmetrical QW, when there are no additional spin–orbit terms: $H_{SO} = 0$. In other words, the Elliot–Yafet mechanism is very important for holes.

In a symmetrical p-type QW, each hole level is doubly degenerate. The wavefunctions have the form

$$\Psi_{sk} = \exp(i\mathbf{k} \cdot \boldsymbol{\rho}) F_s(\mathbf{k}, z) \quad (36)$$

where $s = 1, 2$ enumerates the degenerate states, and Oz is the growth direction.

In a QW based on a III–V semiconductor, the hole wavefunction is a superposition of those four states of the valence band top which correspond to the angular momentum projections on Oz equal to $3/2, 1/2, -1/2$, and $-3/2$. In the basis of these states, $F_s(\mathbf{k}, z)$ are four-component columns which can be represented as [25]

$$F_1 = \begin{bmatrix} -v_0 C(z) \\ i v_1 S(z) e^{i\varphi_k} \\ -v_2 C(z) e^{2i\varphi_k} \\ i v_3 S(z) e^{3i\varphi_k} \end{bmatrix}, \quad F_2 = \begin{bmatrix} i v_3 S(z) e^{-3i\varphi_k} \\ v_2 C(z) e^{-2i\varphi_k} \\ i v_1 S(z) e^{-i\varphi_k} \\ v_0 C(z) \end{bmatrix}. \quad (37)$$

Here $C(z)$ and $S(z)$ are, respectively, symmetrical and asymmetrical functions of the coordinate z , and the real coefficients v_m ($m = 0 \dots 3$) for the ground subband are given by

$$v_m \sim k^m, \quad k \rightarrow 0. \quad (38)$$

An analogue of the electron spin density in experiments on circular polarization of the luminescence in low magnetic fields is the pseudospin of holes, S_h . Its value is proportional to the degree of circular polarization of radiation for relatively weak hole orientation. The hole pseudospin density is given by

$$S_h = \sum_{\mathbf{k}} \text{Tr}[\rho_{\mathbf{k}} \mathbf{J}(\mathbf{k})] \quad (39)$$

where $\mathbf{J}(\mathbf{k})$ is the pseudospin operator for the first hole level of size quantization. In the basis (37), $J^i(\mathbf{k})$ has the form of a 2×2 matrix with the elements

$$J_{ss'}^i(\mathbf{k}) = \int dz F_s^\dagger(\mathbf{k}, z) \mathcal{J}^i F_{s'}(\mathbf{k}, z) \quad (40)$$

where \mathcal{J}^i are 4×4 matrices in the basis of the four states of the valence band top [22]. Calculation yields

$$J^z(E_k) = \sigma_z j_z \quad (41)$$

$$\frac{J^x + iJ^y}{2} = \begin{pmatrix} 0 & a e^{-2i\varphi_k} \\ b e^{4i\varphi_k} & 0 \end{pmatrix} \quad (42)$$

$$\frac{J^x - iJ^y}{2} = \begin{pmatrix} 0 & b e^{-4i\varphi_k} \\ a e^{2i\varphi_k} & 0 \end{pmatrix} \quad (43)$$

where

$$j_z(E_k) = \left(\frac{3}{2}v_0^2 - \frac{1}{2}v_2^2\right) \int dz C^2(z) + \left(\frac{1}{2}v_1^2 - \frac{3}{2}v_3^2\right) \int dz S^2(z) \quad (44)$$

$$a(E_k) = v_1^2 \int dz S^2(z) - \sqrt{3}v_0v_2 \int dz C^2(z) \quad (45)$$

$$b(E_k) = \sqrt{3}v_1v_3 \int dz S^2(z) - v_2^2 \int dz C^2(z). \quad (46)$$

It can be seen that J^z depends on the energy E_k , whereas J^x and J^y depend on both E_k and the direction of \mathbf{k} . Therefore, we have to find not only the isotropic part of the density matrix, $\bar{\rho}$, but also the second and fourth Fourier harmonics.

The J^i have complicated energy dependences. For instance, $j_z(E_k = 0) = 3/2$ for the first heavy-hole subband of size quantization. The dependence of j_z on k is given in [26] for different size-quantization subbands. It follows from equation (38) that $a \sim k^2$ and $b \sim k^4$ for small k , i.e. J^x, J^y are non-zero owing to non-parabolicity effects only.

3.1. Elliot–Yafet mechanism

Let us consider scattering from randomly distributed short-range impurities:

$$V(\mathbf{r}) = \sum_i V_0 \delta(\mathbf{r} - \mathbf{R}_i). \quad (47)$$

Calculating the scattering amplitude $V_{ss'}(\mathbf{k}, \mathbf{k}')$ and using equation (5), we obtain the following equation for the hole density matrix:

$$\frac{\partial \rho_{\mathbf{k}}}{\partial t} = -\frac{2\pi}{\hbar} g(E_k) \oint \frac{d\varphi_{\mathbf{k}'}}{2\pi} (\langle |V_{11}|^2 + |V_{12}|^2 \rangle \rho_{\mathbf{k}} - \langle V_{\mathbf{k}\mathbf{k}'} \rho_{\mathbf{k}'} V_{\mathbf{k}'\mathbf{k}} \rangle). \quad (48)$$

Here, the angular brackets denote averaging over \mathbf{R}_i , and account is taken of the fact that, for (37) and (47),

$$\langle V_{11} V_{12} \rangle = \langle V_{22} V_{12} \rangle = 0$$

and $|V_{ss'}|$ depends on $\theta = \varphi_{\mathbf{k}} - \varphi_{\mathbf{k}'}$. Note that the spin–orbit interaction in p-type QWs is strong and, therefore, the division (6) makes no sense, and we use the total scattering amplitude $V_{\mathbf{k}\mathbf{k}'}$.

We expand the density matrix in a series

$$\rho_{\mathbf{k}} = \sum_{n=-\infty}^{\infty} [\mathcal{I} f_n(E_k) + \boldsymbol{\sigma} \cdot \boldsymbol{\kappa}_n(E_k)] \exp(in\varphi_{\mathbf{k}}). \quad (49)$$

The spin-independent part obeys the equation

$$\frac{\partial f_n}{\partial t} = -\frac{2\pi}{\hbar} g(E_k) \oint \frac{d\theta}{2\pi} \langle |V_{11}|^2 + |V_{12}|^2 \rangle (1 - \cos n\theta) f_n. \quad (50)$$

It is seen that the total number of particles with a given energy $f_0(E_k)$ is conserved, which is correct for the elastic scattering under study. The Fourier harmonics f_n with $n \geq 1$ relax with the corresponding transport times.

For the z -component of $\boldsymbol{\kappa}_n$ we have

$$\frac{\partial \kappa_n^z}{\partial t} = -\frac{2\pi}{\hbar} g(E_k) \oint \frac{d\theta}{2\pi} [\langle |V_{11}|^2 \rangle (1 - \cos n\theta) + \langle |V_{12}|^2 \rangle (1 + \cos n\theta)] \kappa_n^z. \quad (51)$$

It is important that for small k ,

$$\text{Re } V_{11} \gg \text{Im } V_{11} \gg |V_{12}|.$$

Therefore, the first term in square brackets is close to the inverse transport time, and the second is much smaller and describes spin relaxation. It is seen that the average pseudospin with energy E_k , κ_0^z , relaxes over time much more slowly than τ_{tr} . However, higher Fourier harmonics change much faster.

The same is true for $\kappa^{x,y}$. This can be readily shown for small k , when we can neglect V_{12} compared with V_{11} . In this limit

$$\frac{\partial(\kappa_n^x \pm i\kappa_n^y)}{\partial t} = -\Gamma_n^\pm(\kappa_n^x \pm i\kappa_n^y) \quad (52)$$

$$\Gamma_n^\pm = \frac{2\pi}{\hbar} g(E_k) \oint \frac{d\theta}{2\pi} [(|V_{11}|^2)(1 - \cos n\theta) + 2(\text{Im } V_{11})^2 \cos n\theta \mp 2(\text{Re } V_{11} \text{ Im } V_{11}) \sin n\theta]. \quad (53)$$

This means that all harmonics relax with time $\sim \tau_{\text{tr}}$ except that with $n = 0$. The zero-harmonic determines the hole Cooperon, important in the weak-localization effect. Its relaxation is caused by the small term $|V_{12}|$ [27, 28]. However, the in-plane pseudospin components J^x , J^y are determined by the non-zero-harmonics $\kappa_{2,4}^{x,y}$ (see equations (42), (43)) and, hence, their relaxation is fast. Therefore, one can investigate only the dynamics of $S_{h,z}$ in optical experiments. The corresponding equation is

$$\frac{\partial S_{h,z}}{\partial t} = -\frac{4\pi}{\hbar} \frac{\int_0^\infty dE_k g^2(E_k) [F_+(E_k) - F_-(E_k)] j_z(E_k) \oint (d\theta/2\pi) \langle |V_{12}|^2 \rangle}{\int_0^\infty dE_k g(E_k) [F_+(E_k) - F_-(E_k)] j_z(E_k)} S_{h,z}. \quad (54)$$

In [26], the value $(2\pi/\hbar)g(E_k) \oint (d\theta/2\pi) \langle |V_{12}|^2 \rangle$ was calculated as a function of k for different scattering potentials.

3.2. D'yakonov–Perel' mechanism

If we take into account the splitting between the pseudospin states in the valence subbands, which may be caused by both BIA and SIA (in asymmetrical heterostructures), then the operator $H_{\text{SO}} \neq 0$ for holes. For the ground subband, in the basis of the two degenerate states, it is a 2×2 matrix. Therefore, the problem is equivalent to that for electrons, and we can obtain the equation for the density matrix dynamics. Expanding the density matrix in a Fourier series (49), we obtain expressions for f_n and κ_n . The former coincides with equation (50), and the latter is as follows:

$$\frac{\partial \kappa_n^i}{\partial t} = -\frac{2\pi}{\hbar} g(E_k) \oint \frac{d\theta}{2\pi} \tau_n \text{Tr}([H_{-n}, [H_n, \sigma_i]] \sigma_i) \kappa_n^i. \quad (55)$$

According to equations (41)–(43), in order to investigate the hole pseudospin dynamics, we have to average over E_k the equations with $n = 0, \pm 2$, and ± 4 . It can be seen that, similarly to the case for the Elliot–Yafet mechanism, relaxation of $S_{h,x}$ and $S_{h,y}$ is fast, taking a time $\sim \tau_{\text{tr}}$. This is due to the anisotropic nature of the operators J^x and J^y . The only slowly relaxing component is $S_{h,z}$. The corresponding equation has the form

$$\frac{\partial S_{h,z}}{\partial t} = -\frac{1}{2\hbar^2} \sum_{n=-\infty}^{\infty} \frac{\int_0^\infty dE_k g(E_k) [F_+(E_k) - F_-(E_k)] j_z(E_k) \tau_n \text{Tr}([H_{-n}, [H_n, \sigma_z]] \sigma_z)}{\int_0^\infty dE_k g(E_k) [F_+(E_k) - F_-(E_k)] j_z(E_k)} S_{h,z}. \quad (56)$$

In [29], correct equations were obtained for relaxation of the zero-harmonics of the density matrix (for f_0 and κ_0). Therefore, the slow time calculated there describes the relaxation of $S_{h,z}$ only. The relaxation time of $\kappa_0^{x,y}$ was incorrectly associated with that of $S_{h,x}$ and $S_{h,y}$.

To calculate the hole pseudospin relaxation time, knowledge of the concrete form of H_{SO} for the structure being studied is necessary. The BIA contributions can be obtained by averaging the corresponding bulk terms [22]. Moreover, the SIA contributions exist in asymmetrical heterostructures [30]. All of them lead to hole pseudospin relaxation in accordance with equation (56).

4. Conclusions

The predicted spin relaxation anisotropy can be observed in time-resolved measurements similar to those in [21]. In steady-state experiments, the in-plane electron spin relaxation anisotropy can be investigated by means of the Hanle effect. To obtain the spin relaxation times, account should be taken of the fact that the Landé g -factor has not only diagonal in-plane components (g_{xx}), but also off-diagonal ones (g_{xy}) [31]. The degree of photoluminescence polarization in a magnetic field $\mathbf{B} \perp z$ is described by the following expression:

$$P(\mathbf{B}) = \frac{P(0)}{1 + [\mu_B(g_{xx} \pm g_{xy})B/\hbar]^2 \tau_z \tau_{\mp}} \quad (57)$$

where the upper and lower signs correspond, respectively, to the experimental arrangements with $\mathbf{B} \parallel [110]$ and $\mathbf{B} \parallel [\bar{1}\bar{1}0]$ (μ_B is the Bohr magneton). By changing the structure asymmetry (e.g., by applying a gate voltage), one can modify the Hanle curves according to the in-plane spin relaxation anisotropy.

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